

Comments on combinatorial interpretation of Fibonomial coefficients- an e-mail style letter (*)

A.K.Kwaśniewski

High School of Mathematics and Applied Informatics
PL - 15-021 Białystok , ul.Kamienna 17, Poland
e-mail: kwandr@uwb.edu.pl

February 12, 2008

(*) Bulletin of the Institute of Combinatorics and its Applications vol. **42** (2004): 10-11.

Presented also at the Gian-Carlo Rota Polish Seminar
http://ii.uwb.edu.pl/akk/sem/sem_rota.htm

I. Up to our knowledge -since about 126 years we were lacking of classical type combinatorial interpretation of Fibonomial coefficients as it was Lukas [1] - to our knowledge -who was the first who had defined Fibonomial coefficients and derived a recurrence for them (see Historical Note in [2]). Namely as accurately noticed by Knuth and Wilf in [3] the recurrent relations for Fibonomial coefficients appeared already in 1878 Lukas work [1] and in our opinion - Lucas Theorie des Fonctions Numériques Simplement Périodiques is the far more non-accidental context for binomial and binomial-type coefficient - Fibonomial coefficients included.

II. Recently [4] Kwaśniewski combinatorial interpretation of Fibonomial coefficients has been proposed in the spirit [2] of the historically classical standard interpretations according to the schematic correspondences:

SETS —SUBSETS (without and with repetitions)— Binomial coefficient —i.e. we are dealing with LATTICE of subsets

SET PARTITIONS: Stirling numbers of the second kind — number of partitions into exactly k blocs - i.e. we are dealing with LATTICE of partitions.

PERMUTATION PARTITIONS : Stirling numbers of the first kind —

number of permutations containing exactly k CYCLES.

SPACES: q -Gaussian coefficient — number of k -dimensional subspaces in n -th dimensional space over Galois field $GF(q)$ — i.e. we are dealing with LATTICE of subspaces. (For nontrivial and fruitful Konvalina's unified interpretation of the Binomial Coefficients, the Stirling Numbers, and the Gaussian Coefficients see [5]).

POSET — Fibonomial coefficients — number of corresponding (see: [4, 2]) finite "cobweb" subposets of the so called "cobweb" poset.

III. At the time of publishing [4] Kwaśniewski was not aware of the existence of the relevant preprint [6] of Ira M. Gessel and X. G. Viennot (Just few hours ago I have noticed this article via Google) There right after the Theorem 25 (see Section 10 , page 24 in [6]) relating the number $N(R)$ of nonintersecting k -paths to Fibonomial coefficients (via q -weighted type counting formula) the authors express their wish - worthy to be quoted: "*it would be nice to have a more natural interpretation than the one we have given*"..... " *R. Stanley has asked if there is a binomial poset associated with the Fibonomial coefficients...*"

- Well. The cobweb locally finite infinite poset by Kwaśniewski from [2, 4] is not of binomial type. Even more ; the incidence algebra origin arguments seem to make us not to expect binomial type poset come into the game [7]. Am I right?

An immediate question arises - what is the relation like between these two: Gessel and Viennot [6] and [4] points of view? We shall try to elaborate more on that soon.

References

- [1] Eduard Lucas *Thorie des Fonctions Numriques Simplement Priodiques* American Journal of Mathematics **1**(1878) : 184-240 (Translated from the French by Sidney Kravitz , Edited by Douglas Lind Fibonacci Association 1969)
- [2] A.K.Kwasniewski *Combinatorial derivation of the recurrence relation for fibonomial coefficients* ArXiv: math.CO/0403017 v1 1 March 2004
- [3] D. E. Knuth, H. S. Wilf *The Power of a Prime that Divides a Generalized Binomial Coefficient* J. Reine Angev. Math. **396** (1989) : 212-219
- [4] A. K. Kwasniewski *Information on combinatorial interpretation of Fibonomial coefficients* Bull. Soc. Sci. Lett. Lodz Ser. Rech. Deform. 53,

Ser. Rech.Deform. **42** (2003): 39-41 , ArXiv: math.CO/0402291 v1 18 Feb 2004

- [5] John Konvalina *A Unified Interpretation of the Binomial Coefficients, the Stirling Number, and the Gaussian Coefficients* ,Amer. Math. Monthly **245** 107 (2000) : 901-910
- [6] Ira M. Gessel, X. G. Viennot *Determinant Paths and Plane Partitions* preprint (1992) <http://citeseer.nj.nec.com/gessel89determinants.html>
- [7] A.K.Kwasniewski *The second part of on duality triads' paper-On fibonomial and other triangles versus duality triads* Bull. Soc. Sci. Lett. Lodz Ser. Rech. Deform. 53, Ser. Rech. Deform. **42** (2003): 27 -37 ArXiv: math.GM/0402288 v1 18 Feb. 2004